Score: ____/55

3CD MAS Test – May 2010 Calculator Free

1. [4 marks]

4m

Solve |2x + 3| > |x - 1|, showing full reasoning.

2. [3 marks]

3m

Determine the exact value of $\lim_{h\to 0} \left(\frac{\cos 2(a+h) - \cos 2a}{h} \right)$, where a is a constant.

(Om 3. [6 marks]

The equation of one of the tangents to the curve xy(x + y) - 12 = 0 at the points where x = 1 is $y = -\frac{15x}{7} + \frac{36}{7}$.

Determine the equation of the other tangent to the curve when x = 1.

4. [10 marks]

For each of the following functions, find $\frac{dy}{dx}$.

$$3w(a) y = \frac{x^3}{\cos x}$$

(b)
$$y = (\sin x)^x$$

(c)
$$y = \frac{t+2}{t}$$
 and $x = \frac{t-2}{2}$, giving your answer in terms of x .

[4]

[3]

5. [6 marks]

6m

The volume of a cylinder is constant at 50π cm³, but both the height and the radius are changing. Determine the rate at which the radius is changing at the instant when the height is decreasing at a rate of 3 cm/sec and the radius is 5 cm.

6. [9 marks]

Consider the function y = |1 - 2x| + |x|.

(a) Rewrite the function in piecewise form.

Sin

3m

(b) Sketch a graph of the function on the axes provided.

[3]

x

[3]

(c) Hence, differentiate the function with respect to x.

3in

4m 7. [4 marks]

Determine the following limit, showing full reasoning.

$$\lim_{x \to 0} \left(\frac{\tan^2 x}{1 - \cos x} \right)$$

3m 8. [3 marks]

Explain clearly how you would determine the following derivative. (It is not necessary to work out the answer.)

$$\frac{d}{d\left(\sqrt{x}\right)}\ln\!\left[\frac{2\sqrt{x}}{1-\sqrt{x}}\right]$$

9. [10 marks]

Given matrices
$$\mathbf{A} = \begin{bmatrix} 4 & 3 \\ 2 & -1 \end{bmatrix}$$
, $\mathbf{B} = \begin{bmatrix} x & 0 \\ 0 & -1 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} 8 & -3 \\ 5 & 1 \end{bmatrix}$ and $\mathbf{D} = \begin{bmatrix} 2 \\ y \end{bmatrix}$,

- (a) Determine A + D.
- (b) If AB = C, then determine the value of x.

(c) If $\mathbf{A} + 2\mathbf{B} = \begin{bmatrix} 8 & 3 \\ 2 & -3 \end{bmatrix}$, then determine the value of x.

(d) If $x = 2\sqrt{2}$, then determine \mathbf{B}^2 .